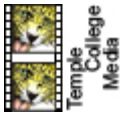
Numbers



After you have read this, the next section will show how to use a calculator to verify your answers.
This and other videos related to numbers and colors are available on the class web site.

Why are there different numbering systems? Just as different countries use different words for the same item, there are different numbering systems that represent the same thing in a different way.

Why are numbers important? Computer only really use 1s and 0s, so from a theoretical point of view, you should be able to convert our human base $_{10}$ numbers to a computer friendly base $_{2}$ number. But from a practical point of view: if you are making web pages, and you don't want to be stuck with 16 word colors, you have to be able to express mixtures of red, green, and blue in numbers. The most widely available graphics program is Paint, but it gives the numerical values of red, green, and blue in base $_{10}$... but web pages use base $_{16}$, hexadecimal. So, just to be able to use a lot of colors, you need to be able to convert the numbers. So, you *should* know the process.

But I know what most of you will do: use a calculator to convert; that's fine, IF you understand the theory.

The three rules of modern numbering systems:
1. Begin with 0 (zero)
2. The base indicates the number of different characters used to represent the numbers (examples base $_{10}$ has 10 characters, base $_{2}$ has 2 characters)
3. When you run out of different characters, add a decimal place to the left, and start over.

Decimal (base 10)		Binary (Base 2)	Hexadecimal (Base 16)
0		0	0
1		1	1
2		10	2
3		11	3
4		**100**	**4**
5		101	5
6		110	6
7		111	7
8		**1000**	**8**
9		1001	9
10		**1010**	**A**

Extended to 16_{10}

Decimal (base 10)		Binary (Base 2)	Hexadecimal (Base 16)
0		0	0
1		1	1
2		10	2
3		11	3
4		**100**	**4**
5		101	5
6		110	6
7		111	7
8		**1000**	**8**
9		1001	9
10		**1010**	**A**
11		1011	B
12		1100	C
13		1101	D
14		1110	E
15		1111	F
16		**10000**	**10**

You should be able to recreate the above chart.
There is a video available to demonstrate counting. The link is below, at the end of the building a 'powers of 2 number line' discussion.

Click class web site for a chart that that goes to 32_{10}, and includes $base_4$ and $base_8$.

Notice how large the binary numbers get; we need a way to represent those numbers that takes up less space, and is more distinctive. If you were to look at the larger example, you can see that base 4, 8, or base 16 are quick ways to represent the same base 2 number with less space. Especially base 16. But decimal, base 10, does not easily fit in this pattern.

Let's find out why.

Look at the numbers in the binary column. Notice the numbers that begin with one, and are followed by nothing or zeros… and compare those to the decimal equivalent.

1	10	100	1000	10000	100000
1	2	4	8	16	32

The decimal values are doubling. This becomes the basis of a quick to build **powers-of-two number line**.

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1

There is a video available on counting, and the Powers of 2 number line here.

This number line is a quick key to converting decimal to binary, and binary to decimal; this is the hardest conversion as the number 10 does not occur on the powers of two number line.

The Harder Stuff, part 1: Converting decimal to binary.
After you have read this, you may wish to go to p. 134 see how to use a calculator to verify your answers.

115_{10} converts to $?_2$

1. Always build a powers-of-two number line. You don't need to calculate it, just start with 1, then double!
2. Notice the subscript is getting smaller… that is our clue to 'subtract'.
3. Computers use 1 to represent success (yes) and 0 to represent failure (no). We will always write our successes and failures (1s and 0s) under the powers-of-two number line.

The largest number on the powers-of-two number line we can **subtract** from 115 is 64. Put a one under the 64 to represent "yes, I can subtract this," then do the math… subtract 115-64= 51

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
				1						

Can we subtract the next number to the right from 51? Yes! Record the success with a 1, and do the math. 51-32 =19

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
				1	1					

Can we subtract the next number to the right from 19? Yes! Record the success with a 1, and do the math. 19-16 =3

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
				1	1	1				

Can we subtract the next number to the right from 3? No! Record the failure with a 0, and continue.

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
				1	1	1	0			

Can we subtract the next number to the right from 3? No! Record the failure with a 0, and continue.

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
				1	1	1	0	0		

Can we subtract the next number to the right from 3? Yes! Record the success with a 1, and do the math. 3-2 =1

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
				1	1	1	0	0	1	

Can we subtract the next number to the right from 1? Yes! Record the success with a 1, and do the math. 1-1 =0

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
				1	1	1	0	0	1	1

There are no more subtractions possible, but we have our number! 115_{10} converts to 1110011_2

Note: since 115 is odd, it must end in 1.
After you have read this, you may wish to go to p. 134 see how to use a calculator to verify your answers.

The Harder Stuff, part 2: Converting binary to decimal.
After you have read this, you may wish to go to p. 134 see how to use a calculator to verify your answers.

11101_2 converts to $?_{10}$

1. Always build a powers-of-two number line. You don't need to calculate it, just start with 1, then double!
2. Notice the subscript is getting larger… that is our clue to 'add'.
3. Computers use 1 to represent success (yes) and 0 to represent failure (no). We will always write our successes and failures (1s and 0s) under the powers-of-two number line.

Write the 1s and 0s under the powers-of-two number line.

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
						1	1	1	0	1

Add the value **above** the successes (1s) and ignore the failures (0s).

16+8+4+1 = 29

We have our number! 11101_2 converts to 29_{10}
After you have read this, you may wish to go to p. 134 see how to use a calculator to verify your answers.

The Really Easy Stuff, part 1: Binary to Octal, see class website

The Really Easy Stuff, part 2: Binary to Hexadecimal
After you have read this, there is a section on how to use a calculator to verify your answers.

110001011101_2 converts to $?_{16}$

Divide the number into blocks of four, starting from the right. If you don't have enough numbers, add 0s to the left.

1100 0101 1101

Put a short powers-of-two number line above each of the blocks of four; remember, the 1s and 0s always go under the powers-of-two number line.

8 4 2 1	8 4 2 1	8 4 2 1
1 1 0 0	0 1 0 1	1 1 0 1
12	5	13
12 in hex from the chart , C	5 in hex from the chart , 5	13 in hex from the chart , D

There's our number! 110001011101_2 converts to $C5D_{16}$
After you have read this, you may wish to go to p. 134 see how to use a calculator to verify your answers..

Hexadecimal, a BAD numbering system, to Binary

BAD_{16}

B	A	D
11 in hex from the chart or	10 in hex from the chart or	13 in hex from the chart or
1011	1010	1101

$1011\ 1010\ 1101_{\ 2}$
After you have read this, you may wish to go to p. 134 see how to use a calculator to verify your answers.

Just for jollies, Hexadecimal, a BAD numbering system, to Decimal (just use all the skills he have learned)

BAD_{16}

B	A	D
11 in hex from the chart or	10 in hex from the chart or	13 in hex from the chart or
1011 in binary from chart of conversion	1010 in binary from chart of conversion	1101 in binary from chart of conversion

Put 101110101101 under a powers-of-two number line, add the successes to get 298910!

PRACTICE QUESTIONS

Converting decimal to binary.

13_{10} converts to $?_2$

255_{10} converts to $?_2$

5_{10} converts to $?_2$

Converting binary to decimal.

1011_2 converts to $?_{10}$

111_2 converts to $?_{10}$

111010_2 converts to $?_{10}$

Converting hexadecimal to binary.

25_{16} converts to $?_2$

$1F_{16}$ converts to $?_2$

$F1_{16}$ converts to $?_2$

Converting hexadecimal to decimal.

25_{16} converts to $?_{10}$

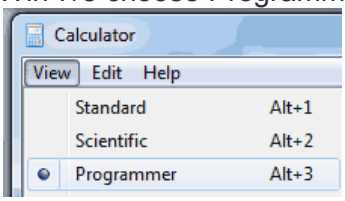
$1F_{16}$ converts to $?_{10}$

$F1_{16}$ converts to $?_{10}$

You might want to check your answers with the Windows calculator.

Class website has videos on Starting the Windows Calculator/Using the Windows Calculator

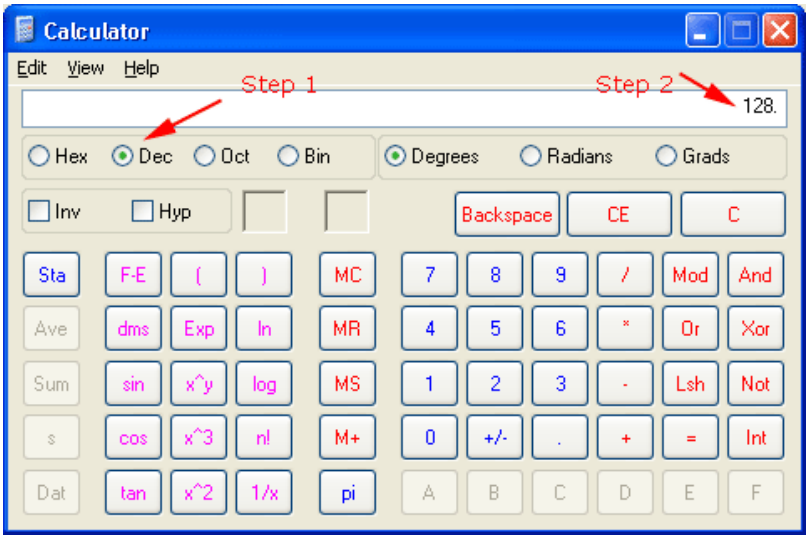
If using Win XP, go to View menu, and choose Scientific View. Win 7/8 choose Programmer

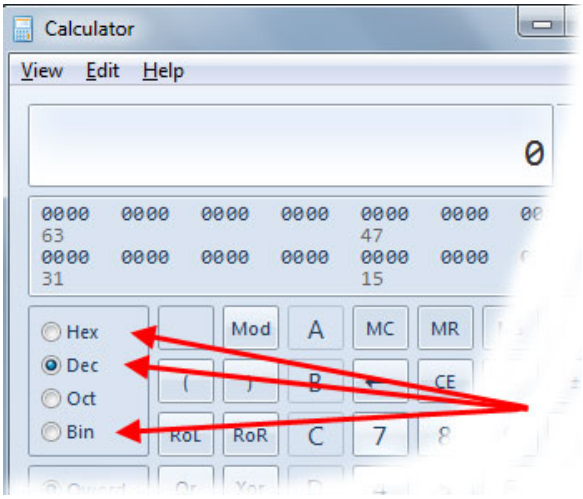


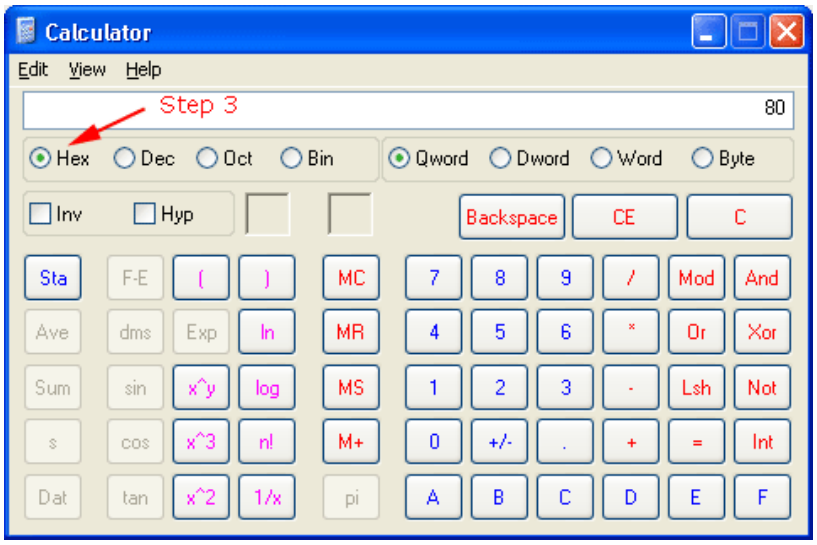
Mac: Macintosh HD\Applications\Calculator and then go to View ▸ Programmer.

1) If the starting number is in base $_{10}$, set the button to Dec (if base $_{2}$ Bin, if base $_{16}$ Hex)
2) enter the number to convert,
3) then click to Bin, Dec, or Hex to convert and display the number. It's that easy

Example $128_{10}=?_{16}$ (in Win 7 the Hex, Dec, and Bin buttons are on the side, all else same)







1) If the starting number is in base $_{10}$, set the button to Dec (if base $_{2}$ Bin, if base $_{16}$ Hex)
2) enter the number to convert,
3) then click to Bin, Dec, or Hex to convert and display the number. It's that easy

Exponents
If you are trying to solve something like $2^8*2^8*2^8=2^{24}$, and you want to know what 2^{24} is,
Type in 2
then click the x^y button
then type the exponent, 24
then press =, or hit the enter key to calculate

Notes